Z_2 fractionalized phases of t-J models

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May 4, 2018
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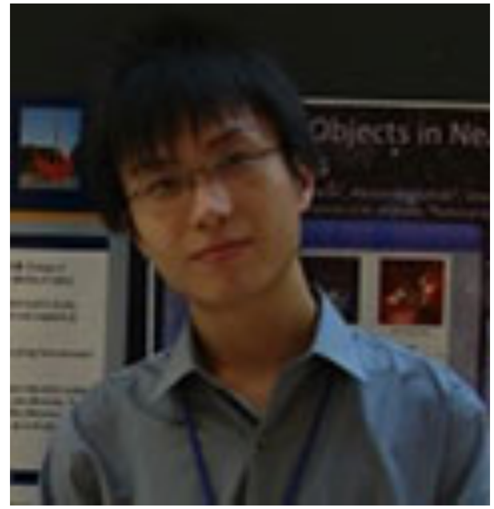
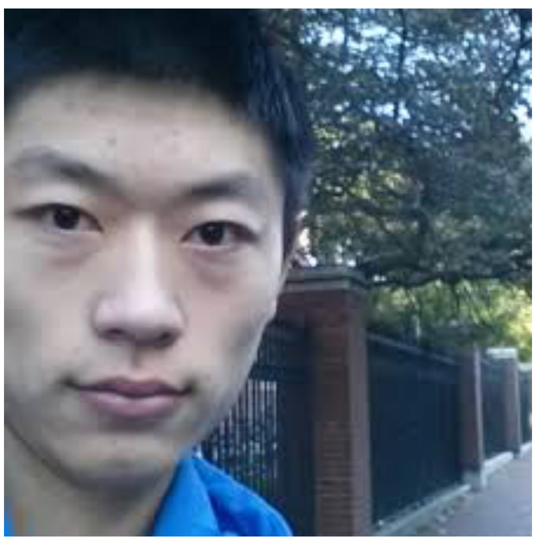


Chong Wang
Harvard

Ashvin Vishwanath
Harvard

arXiv:1804.01095



Wenbo Fu
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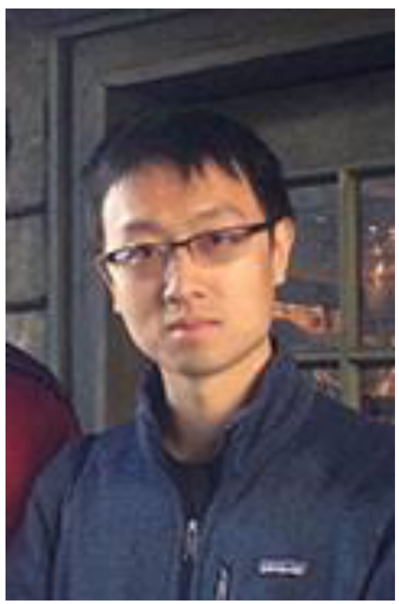

Yingfei Gu
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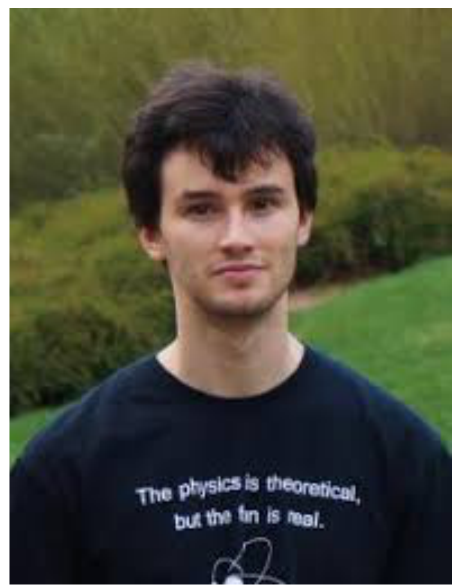

Grisha Tarnopolsky
Harvard

arXiv:1804.04130

1. Orthogonal metals

2. Confinement transitions in square lattice Z_2 gauge theories: CFTs with emergent gauge fields

3. Phase diagram of a SYK model

1. Orthogonal metals

2. Confinement transitions in square lattice Z_2 gauge theories: CFTs with emergent gauge fields

3. Phase diagram of a SYK model

Orthogonal metals

Fractionalize the electron $c_{i\alpha}$, $\alpha =\uparrow,\downarrow$ into an "orthogonal fermion" $f_{i\alpha}$ and an Ising spin $\sigma_i^z = \pm 1$:

$$c_{i\alpha} = \sigma_i^z \, f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\sigma_i^z \to \eta_i \sigma_i^z \quad , \quad f_{i\alpha} \to \eta_i f_{i\alpha}$$

The orthogonal fermion, f_α, carries both the spin and charge of the electron.
The Ising matter field, σ^z, is 'dark matter' carrying only energy, and a $\mathbb{Z}_2$ gauge charge.

R. Nandkishore, M. Metlitski, and T. Senthil, Phys. Rev. B 86, 045128 (2012)

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2. Confinement transitions in square lattice Z_2 gauge theories: CFTs with emergent gauge fields

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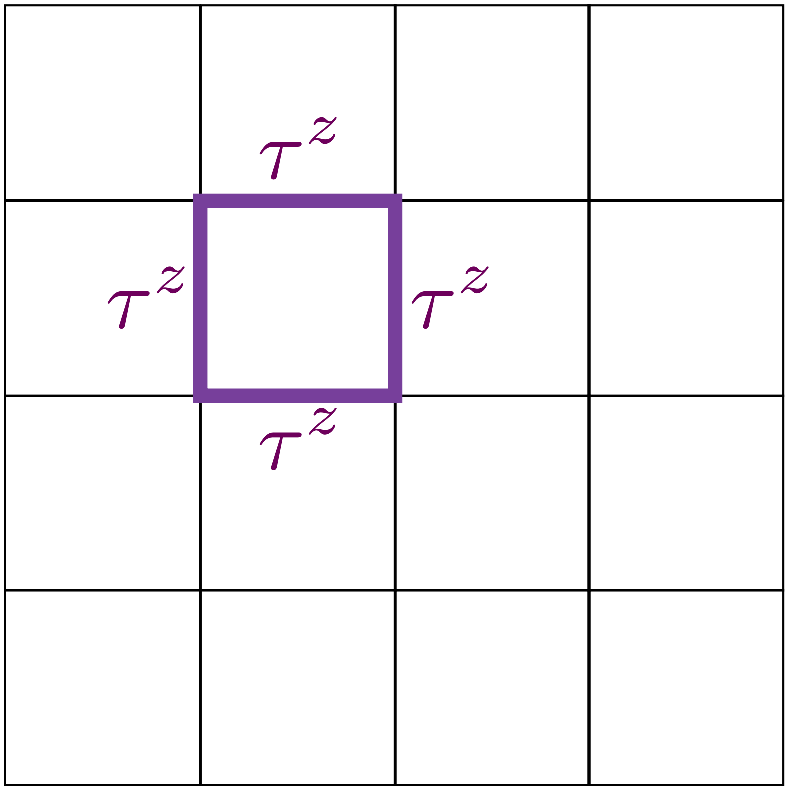
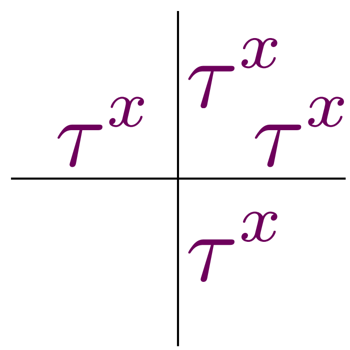
Z_2 lattice gauge theory (Wegner, 1971)

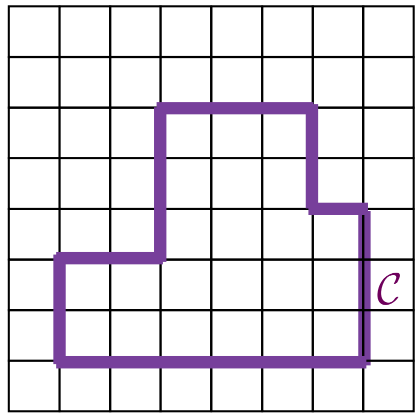


$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

$G_i =$


Gauss's Law: $[H, G_i] = 0 \quad , \quad G_i = 1$

Z_2 lattice gauge theory (Wegner, 1971)



$$W_{\mathcal{C}} = \prod_{\mathcal{C}} \tau^z$$

Deconfined phase.
'Perimeter law' for
Wegner-Wilson loops

Confined phase.
'Area law' for
Wegner-Wilson loops

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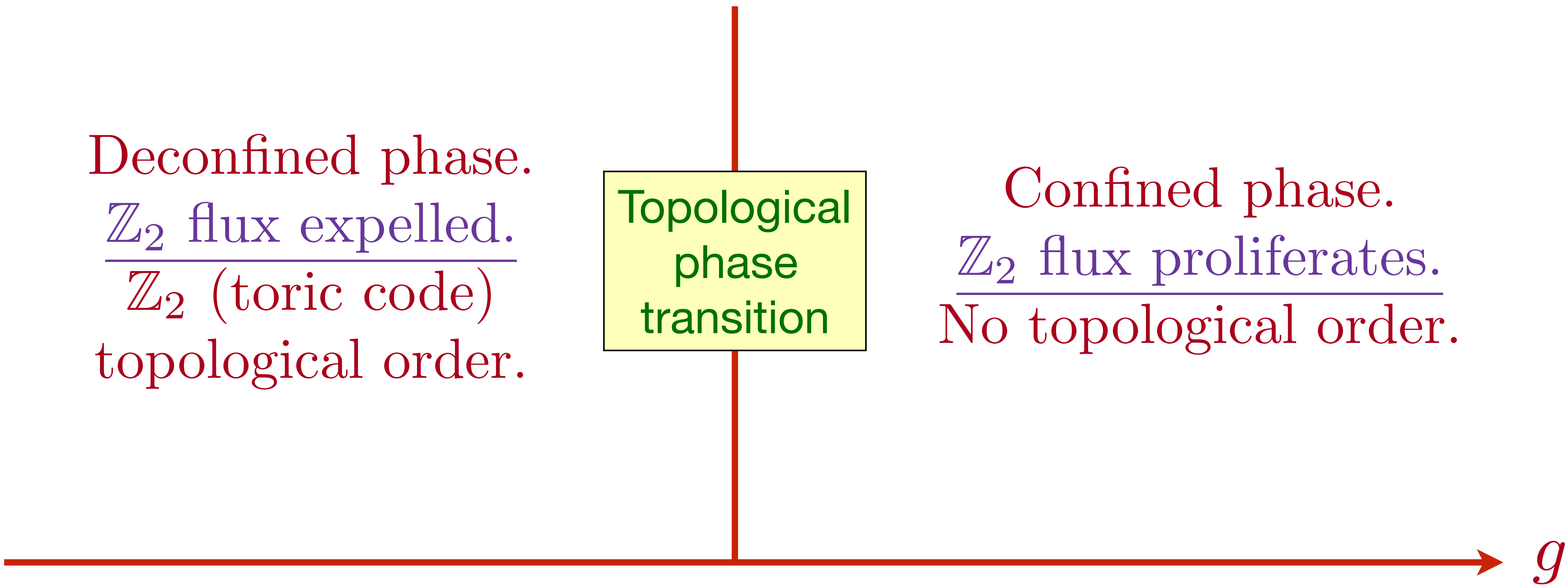
$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

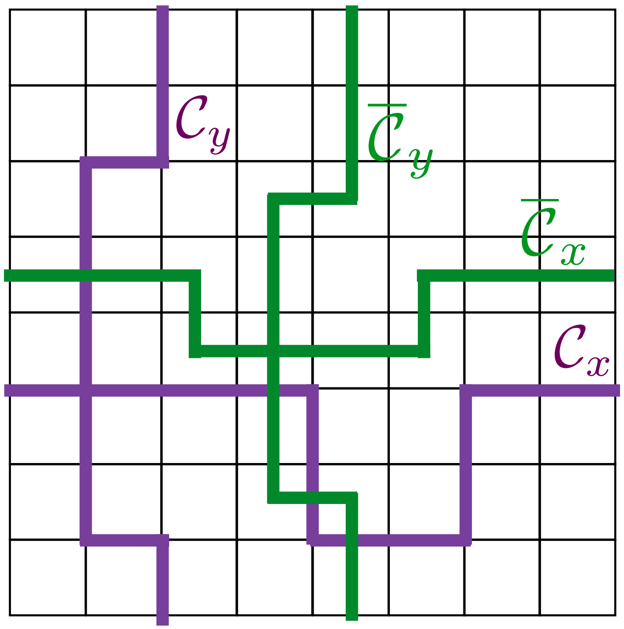
$\mathbb{Z}_2$ lattice gauge theory



$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x$$

E. Fradkin and S. H. Shenker, PRD **19**, 3682 (1979); N. Read and S. Sachdev, PRL **66**, 1773 (1991); X.-G. Wen, PRB **44**, 2664 (1991); A.Y. Kitaev, Annals of Physics **303**, 2 (2003)

Topological order



$$V_x = \prod_{\bar{\mathcal{C}}_x} \tau^x \quad , \quad V_y = \prod_{\bar{\mathcal{C}}_y} \tau^x$$

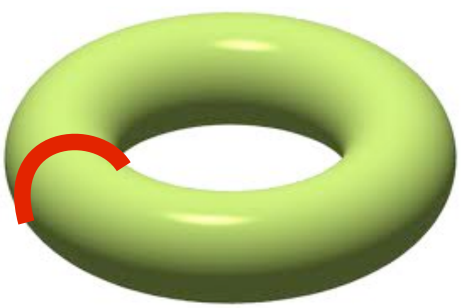
$$W_x = \prod_{\mathcal{C}_x} \tau^z \quad , \quad W_y = \prod_{\mathcal{C}_y} \tau^z$$

$$V_x W_y = -W_y V_x \quad , \quad V_y W_x = -W_x V_y$$

and all other pairs commute.

On a torus, there are two additional independent operators, V_x and V_y which commute with the Hamiltonian:

$$[H, V_x] = [H, V_y] = 0$$



(N. Read and S.S., 1991 Freedman, Nayak, Shtengel, Walker, Wang, 2003)

Topological quantum field theory describes degenerate states with Z_2 flux $W = \pm 1$ through the holes of the torus

Confined phase.
Unique ground state has $V_x = 1$, $V_y = 1$.
No topological order

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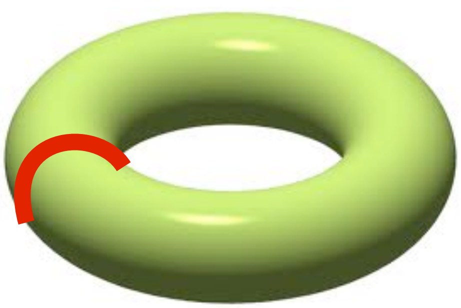
This criterion can distinguish the phases when dynamical (or even gapless) matter fields are present

Z_2 lattice gauge theory

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \quad , \quad \boxed{G_i = 1}$$

$$\mathcal{L} = |(\partial_\mu - 2ia_\mu)\Phi|^2 + |\Phi|^4$$
$$+ \text{ relevant monopoles.}$$
Ising* criticality

(Fradkin and Shenker, 1979)



Higgs state with $\langle \Phi \rangle \neq 0$: The phase of Φ winds by 2π around the cycle of the torus, trapping U(1) flux π in the hole of the torus. This leads to 4-fold degeneracy

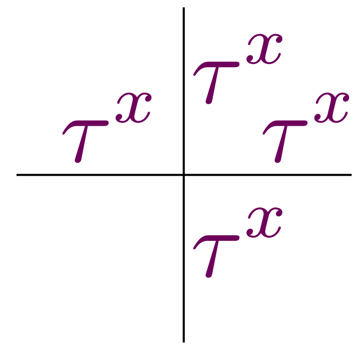
Confined phase. Unique ground state has $V_x = 1$, $V_y = 1$. No topological order

g

(N. Read and S.S., 1991; X.-G. Wen, 1991
T. Senthil and M.P.A. Fisher, 2000
J. Maldacena, G. Moore, N. Seiberg, 2001)

Odd Z_2 lattice gauge theory

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \quad , \quad \boxed{G_i = -1}$$

$G_i =$ 

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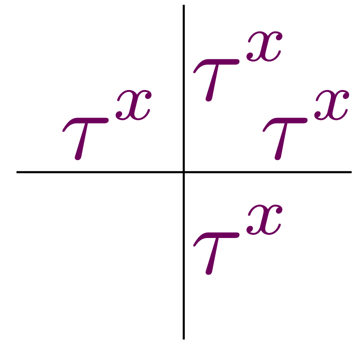
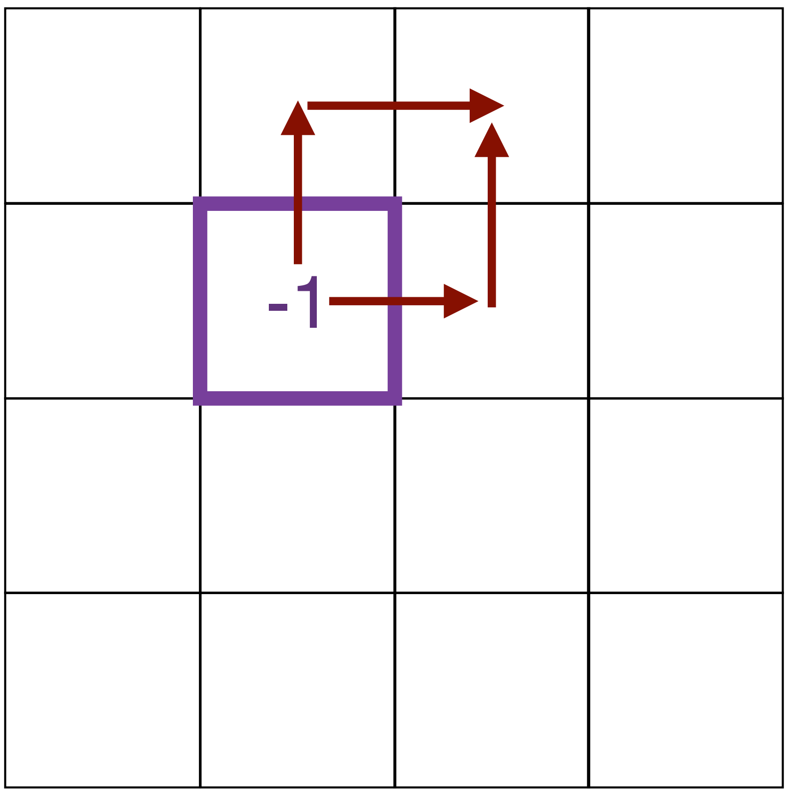
$G_i =$ 



Symmetry fractionalization:

Single spacing translations anti-commute

$$T_x T_y = -T_y T_x$$

when acting on
'fractionalized' states with Z_2 flux -1.

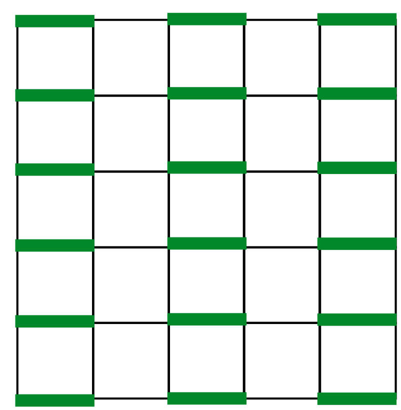
Odd Z_2 lattice gauge theory

$$H = -\sum_{\square} \tau^z \tau^z \tau^z \tau^z - g \sum_i \tau^x \quad , \quad \boxed{G_i = -1}$$

Trivial phase is prohibited

Deconfined phase.
Topological order
Particles with $\mathbb{Z}_2$ flux have a degenerate spectrum which realizes
$T_x T_y = -T_y T_x$

Confined phase.
Broken symmetry and valence bond solid (VBS) order



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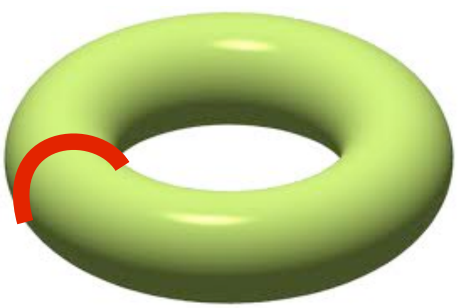
Deconfined quantum criticality:
$$\mathcal{L} = |(\partial_\mu - 2ia_\mu)\Phi|^2 + |\Phi|^4$$
+ irrelevant quadrupled monopoles



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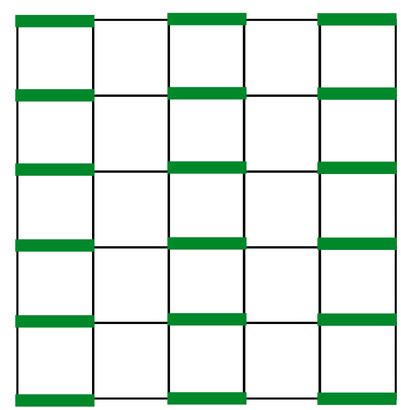
Trivial phase is prohibited

Confined phase. Broken symmetry and valence bond solid (VBS) order



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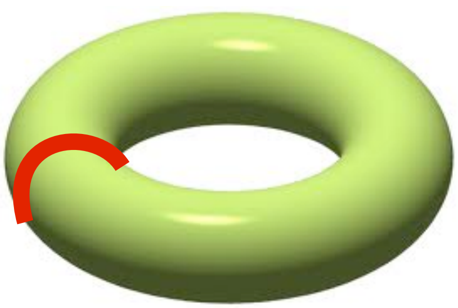
Trivial phase is prohibited

Broken symmetry of the massless scalar dual to the photon

Confined phase. Broken symmetry and valence bond solid (VBS) order



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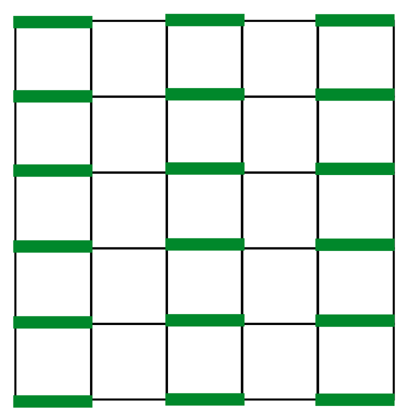

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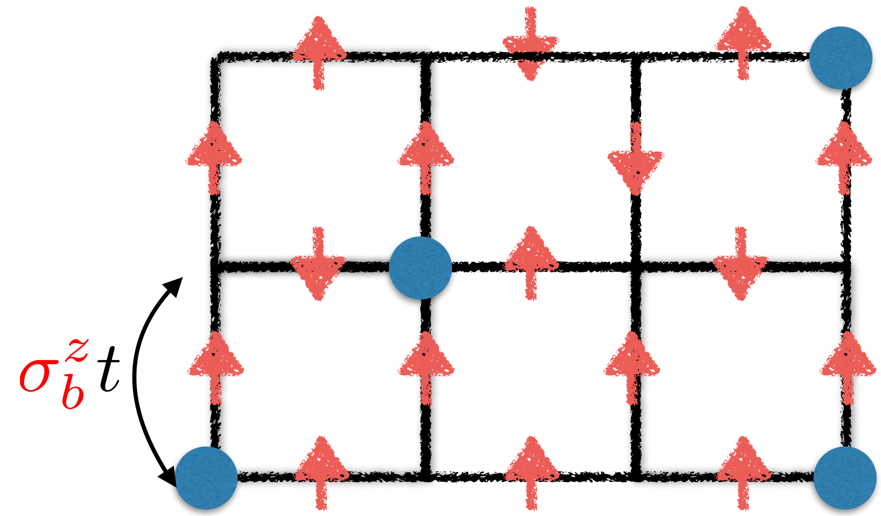
Z_2 gauge theory of orthogonal fermions

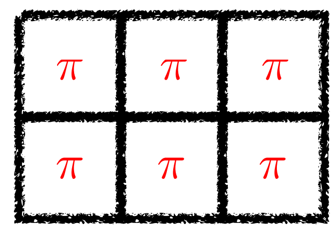
Grover, Assaad (2016)
Gazit, Randeria, Vishwanath (2017)

Ising Gauge field σ_b^z

fermion $f_{i,\alpha}$



$$\mathcal{H}_{\mathbb{Z}_2} = +|J| \sum_{\square} \prod_{b\in\square} \sigma_b^z - h \sum_b \sigma_b^x$$



$J \gg h$

π-flux

$$\mathcal{H}_f = -t \sum_{b=\langle i,j\rangle,\alpha} \sigma_b^z f_{i,\alpha}^\dagger f_{j,\alpha} + h.c$$

Ising "Gauss law" with matter fields

$$G_r = (-1)^{n_r^f} \prod_{b\in r} \sigma_b^x = -1$$ "Odd" Lattice gauge theory

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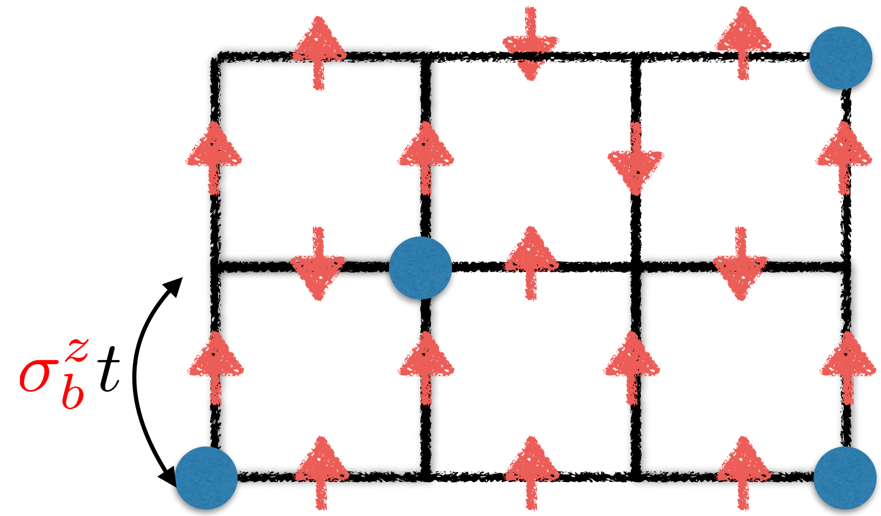

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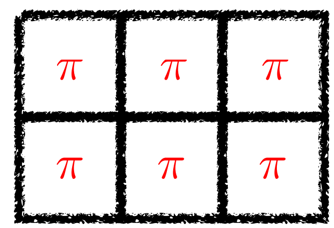


$J \gg h$

π-flux

Global Symmetries

$$SU_s(2) \times SU_c(2)$$

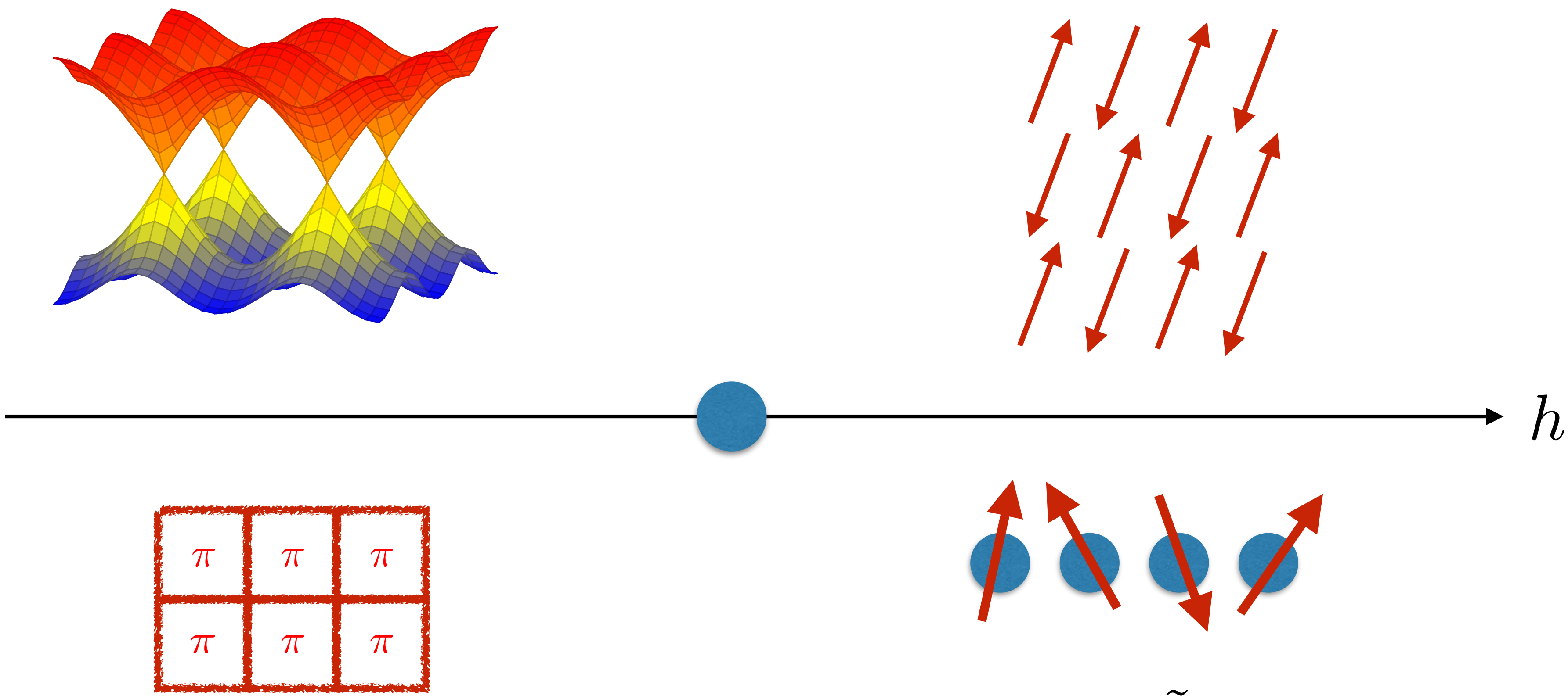
Spin rotations

Pseudo-Spin BCS/CDW

Phase Diagram

Deconfined Dirac

Confined AFM



$$\mathcal{H}_{\text{eff}} \sim \tilde{J}\mathbf{S}_i \cdot \mathbf{S}_j$$

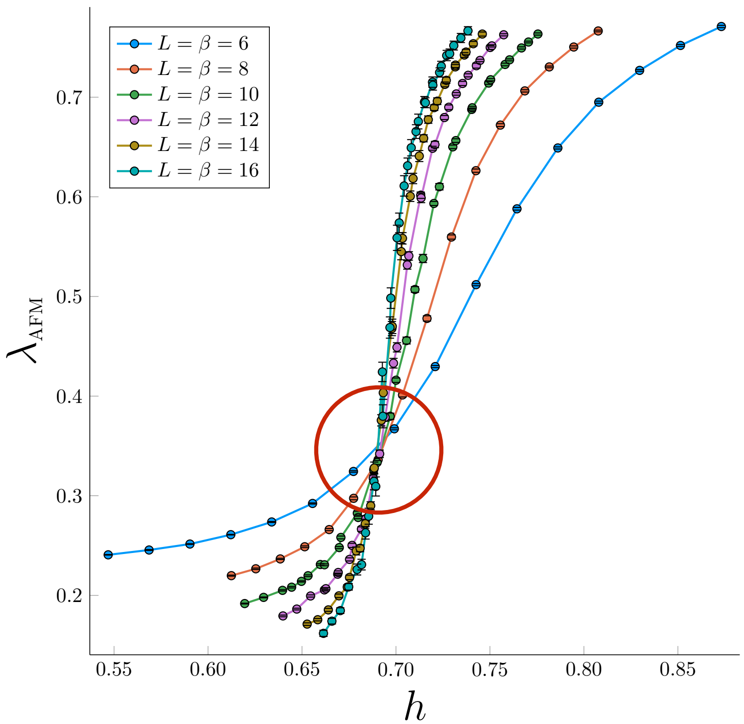
π-flux

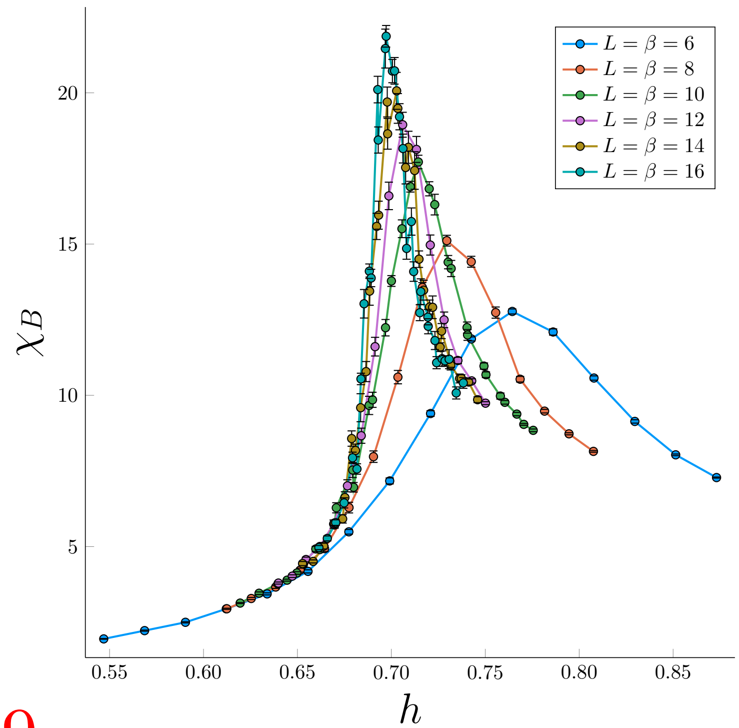
AFM order

AFM + confinement transition

$$\lambda_{\mathrm{AFM}} = 1 - \frac{\chi_{\mathbf{S}}\left(G_{\mathrm{AFM}} - \Delta q\right)}{\chi_{\mathbf{S}}\left(G_{\mathrm{AFM}}\right)}$$

$$\chi_B = \langle B^2 \rangle - \langle B \rangle^2$$





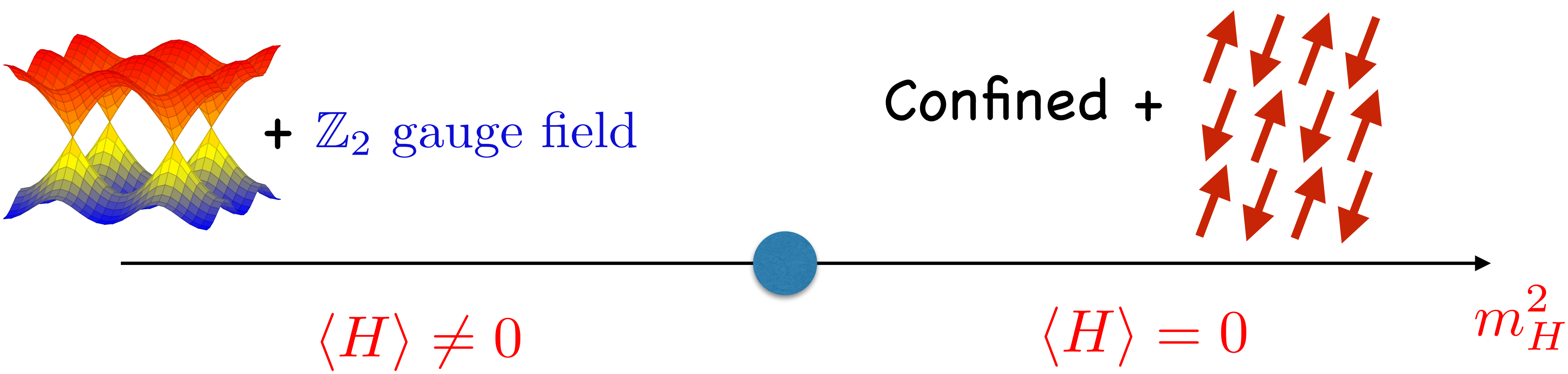
$$h_c = 0.69$$

Symmetry breaking and confinement coincide w/o fine-tuning!

Critical Field theory

- Embed the Ising gauge theory as $\mathbb{Z}_2 \in SU_g(2)$
- Introduce a 3×3 real matrix field H_{ab} which transforms under $SU_g(2)$ (left multiplication) and $SU_c(2)$ (right multiplication).

Kartik (2018)



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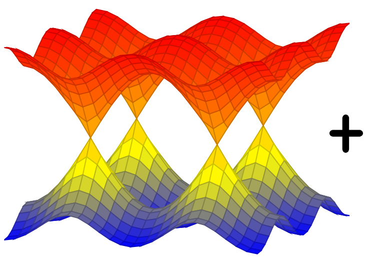
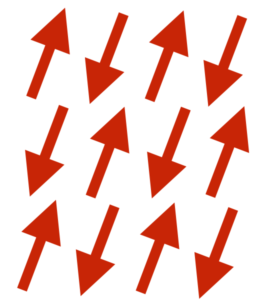


+ $\mathbb{Z}_2$ gauge field

Confined +



$\langle H \rangle \neq 0$ $\qquad$ $\langle H \rangle = 0$ $\qquad$ m_H^2

Critical theory is $N_f = 2$ SU(2) QCD_3 and a critical Higgs field H

- Higgs phase, $\langle H \rangle \neq 0$ has $\mathbb{Z}_2$ topological order and massless Dirac orthogonal fermions
- When the Higgs field is gapped, QCD_3 confines, leading to AFM order.

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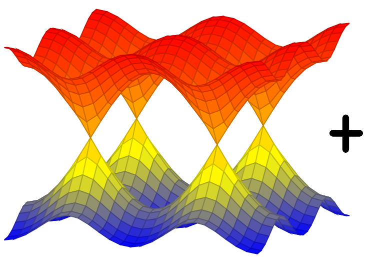

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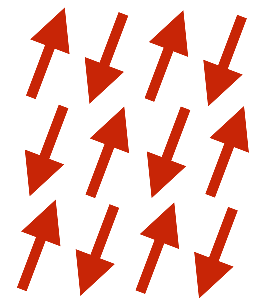


$\langle H \rangle \neq 0$ $\langle H \rangle = 0$ m_H^2

Prediction - $N_f = 2$ SU(2) QCD_3 has an emergent SO(5) global symmetry

Evidence for SO(5) symmetry

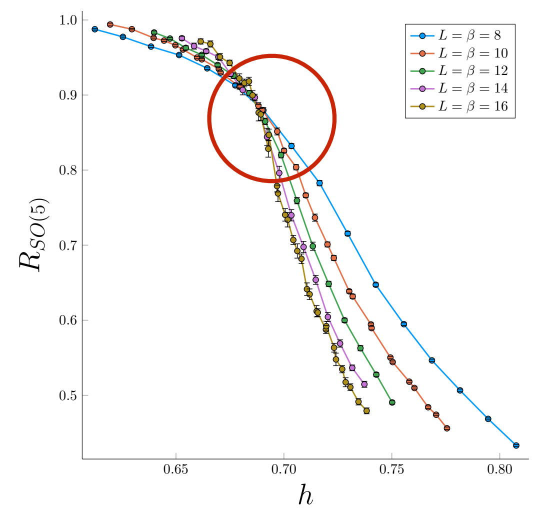
$$\mathbf{B}^{\eta}(q) = \sum_{r,\alpha} e^{iq\cdot r}\left(\sigma^{z}_{r,\eta} f^{\dagger}_{r+\eta,\alpha} f_{r,\alpha} + \mathrm{h.c}\right)$$

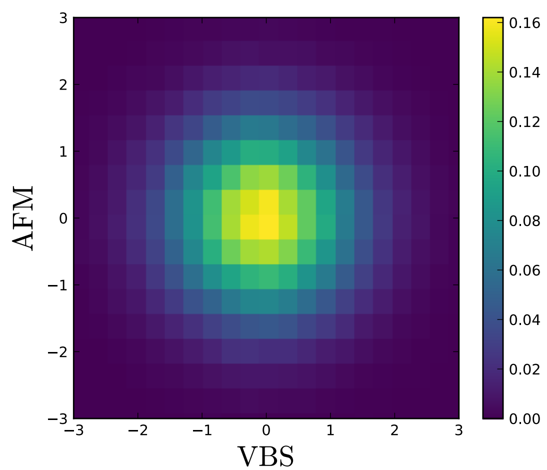
$$\mathbf{S}^{\gamma}(q) = \sum_{r,\alpha,\beta} e^{iq\cdot r} f^{\dagger}_{r,\alpha} \tau^{\gamma}_{\alpha\beta} f_{r,\beta}$$

$$\begin{pmatrix} \mathbf{B}^{x} \\ \mathbf{B}^{y} \\ \mathbf{S}^{x} \\ \mathbf{S}^{y} \\ \mathbf{S}^{z} \end{pmatrix} \begin{array}{l} \updownarrow \text{VBS} \\ \\ \updownarrow \text{AFM} \end{array}$$

RG invariant $R_{SO(5)} = \dfrac{\chi_{\mathrm{VBS}}}{\chi_{\mathrm{AFM}}}$

Curve crossing at $h_c = 0.69$





$$\mathbf{P}(\mathbf{B}^{x}, \mathbf{S}^{z})$$

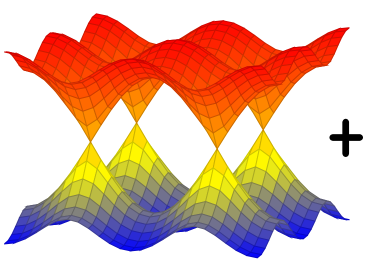
At criticality displays circular symmetry

Critical Field theory

1. Embed the Ising gauge theory in $\mathbb{Z}_2 \in SU_g(2)$

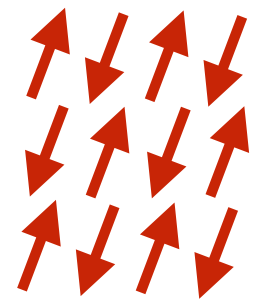
2. 3x3 matrix Higgs field H_{ab} transforms as spin-one (SO(3)) under $SU_g(2)$

Kartik (2018)



+ $SU(2)/SO(3) = \mathbb{Z}_2$

Confined +



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2. Confinement transitions in square lattice Z_2 gauge theories: CFTs with emergent gauge fields

3. Phase diagram of a SYK model

Orthogonal metals

Fractionalize the electron $c_{i\alpha}$, $\alpha =\uparrow,\downarrow$ into an "orthogonal fermion" $f_{i\alpha}$ and an Ising spin $\sigma_i^z = \pm 1$:

$$c_{i\alpha} = \sigma_i^z \, f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\sigma_i^z \rightarrow \eta_i \sigma_i^z \quad , \quad f_{i\alpha} \rightarrow \eta_i f_{i\alpha}$$

The orthogonal fermion, f_α, carries both the spin and charge of the electron.
The Ising matter field, σ^z, is 'dark matter' carrying only energy, and a $\mathbb{Z}_2$ gauge charge.

R. Nandkishore, M. Metlitski, and T. Senthil, Phys. Rev. B 86, 045128 (2012)

Orthogonal metals

Fractionalize the electron $c_{ip\alpha}$, on sites $i = 1 \ldots N$, with spin $\alpha = 1 \ldots M$ and orbital index $p = 1 \ldots M'$ into an "orthogonal fermion" $f_{i\alpha}$ and a real scalar ϕ_{ip}:

$$c_{ip\alpha} = \phi_{ip} \, f_{i\alpha}$$

This introduces a $\mathbb{Z}_2$ gauge invariance

$$\phi_{ip} \to \eta_i \phi_{ip} \quad , \quad f_{i\alpha} \to \eta_i f_{i\alpha}$$

The orthogonal fermion $f_{i\alpha}$ carries both the spin and charge of the electron.
The scalar field, ϕ_{ip}, is 'dark matter' carrying only energy, and a $\mathbb{Z}_2$ gauge charge.

A solvable model

We examine the t-J model:

$$\begin{aligned}\mathcal{L} &= \frac{1}{2g}\sum_{i,p}\left(\partial_\tau \phi_{ip}\right)^2 + \sum_{i,\alpha} f_{i\alpha}^\dagger\left(\frac{\partial}{\partial\tau} - \mu\right) f_{i\alpha} \\ &\quad + \frac{1}{\sqrt{NM}}\sum_{i,j,p,\alpha} t_{ij}\phi_{ip}\phi_{jp} f_{i\alpha}^\dagger f_{j\alpha} + \frac{1}{\sqrt{NM}}\sum_{i>j,\alpha\beta} J_{ij} f_{i\alpha}^\dagger f_{i\beta} f_{j\beta}^\dagger f_{j\alpha}\,,\end{aligned}$$

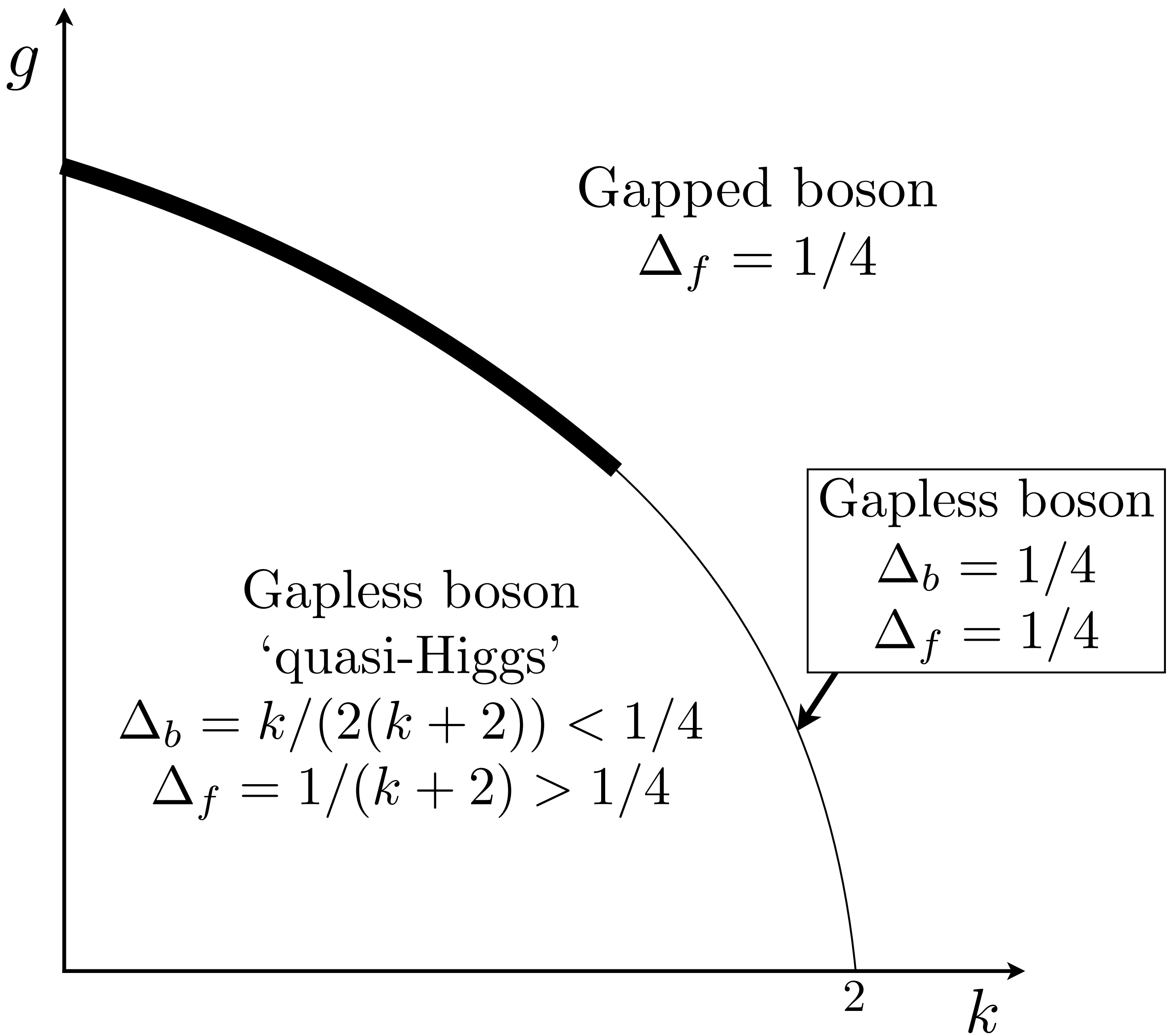
with the scalar field obeying the fixed length constraint

$$\sum_{p=1}^{M'} \phi_{ip}^2 = M'\,.$$

With t_{ij} and J_{ij} independent random numbers with zero mean, $\mathcal{L}$ is solvable in the limit of large number of sites, N, followed by the limit of large M and M' at fixed

$$k \equiv \frac{M'}{M}\,.$$

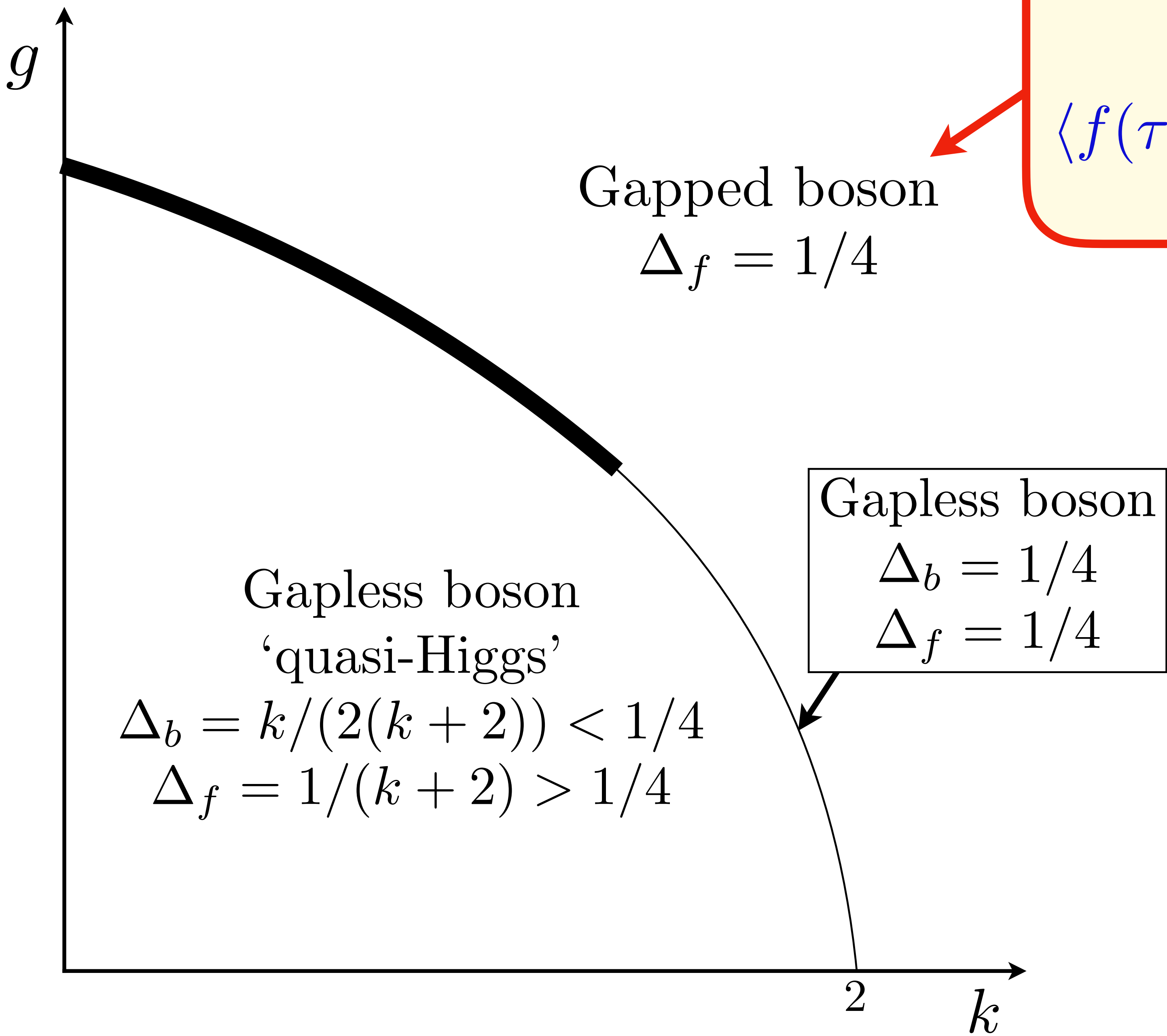
A solvable model



A solvable model

$$\langle \phi(\tau)\phi(0)\rangle \sim \frac{e^{-m|\tau|}}{\sqrt{\tau}}$$

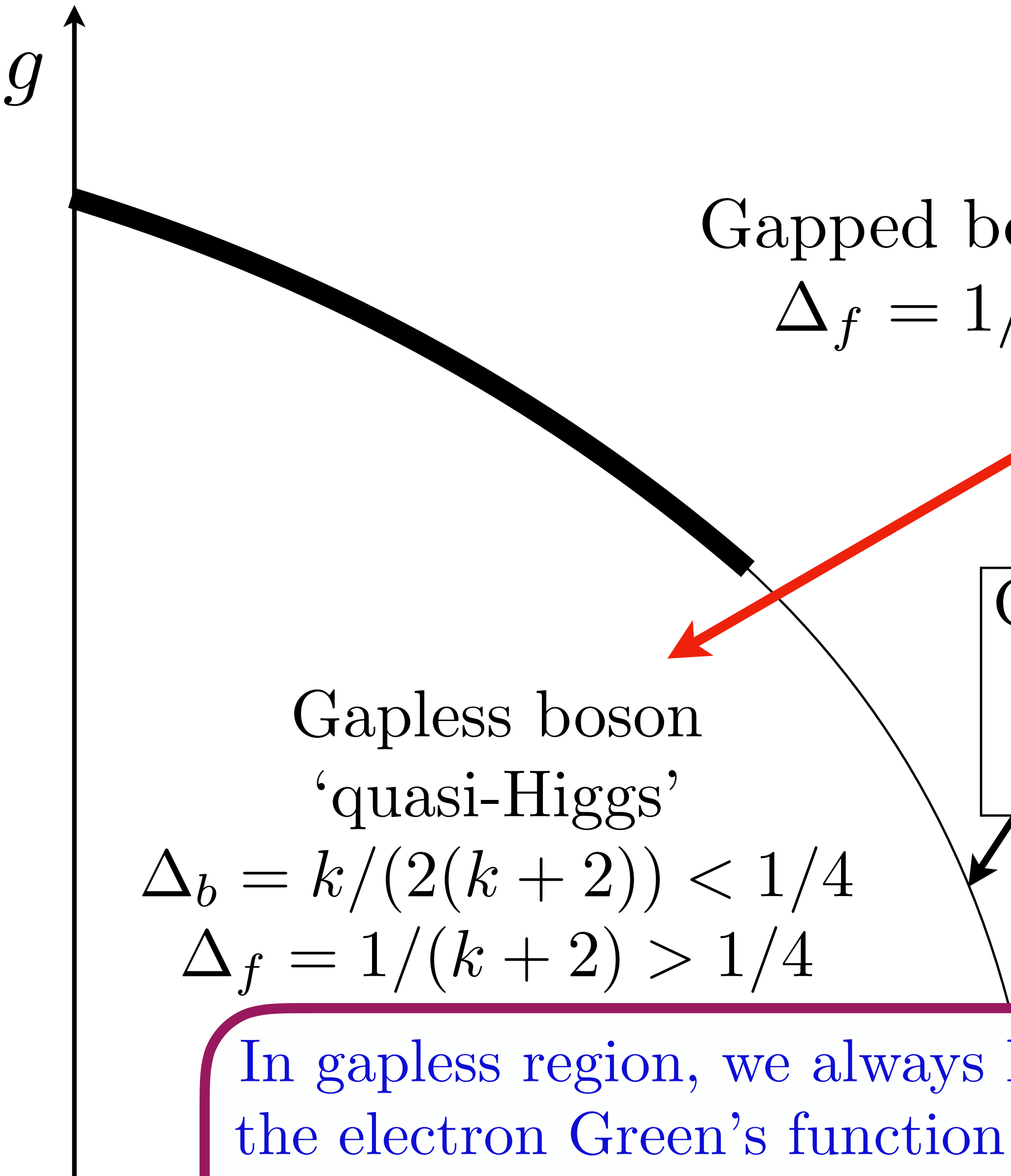
$$\langle f(\tau) f^\dagger(0)\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{2\Delta_f}}$$



A solvable model

$$\langle \phi(\tau)\phi(0)\rangle \sim \frac{1}{|\tau|^{2\Delta_b}}$$

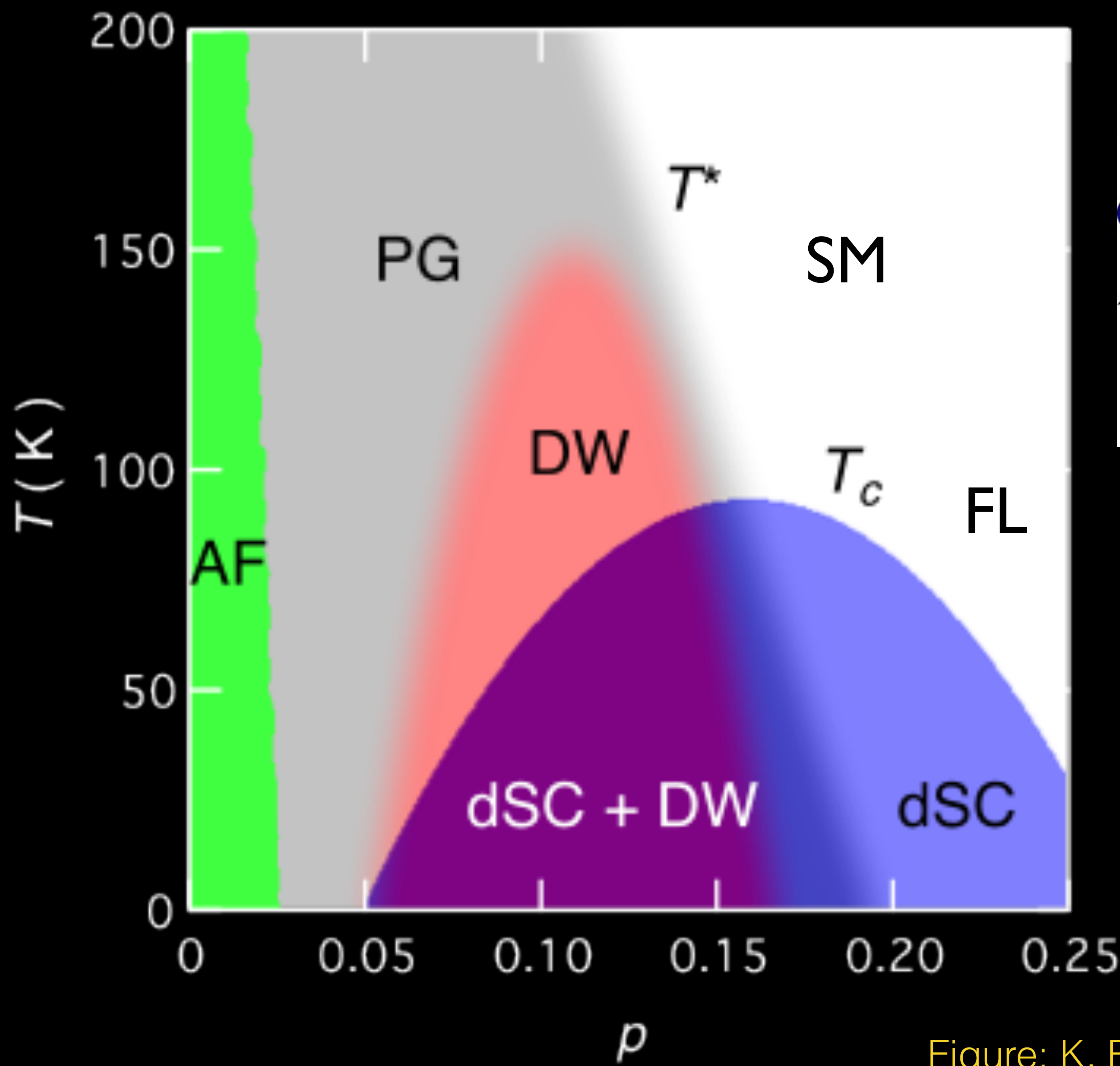
$$\langle f(\tau)f^\dagger(0)\rangle \sim \frac{\mathrm{sgn}(\tau)}{|\tau|^{2\Delta_f}}$$

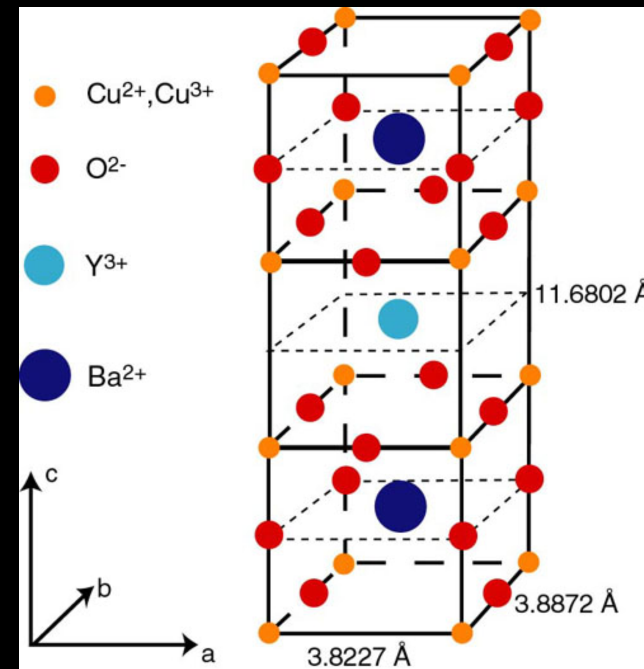


Gapless boson
$\Delta_b = 1/4$
$\Delta_f = 1/4$

$$\Delta_b + \Delta_f = 1/2$$

In gapless region, we always have the Fermi liquid form for the electron Green's function $\langle c(\tau)c(0)\rangle \sim 1/\tau$, although $\mathbb{Z}_2$ charges remain deconfined. This is a consequences of 'balancing' the t and J terms in the t-J model.





$YBa_2Cu_3O_{6+x}$

Figure: K. Fujita and J. C. Seamus Davis

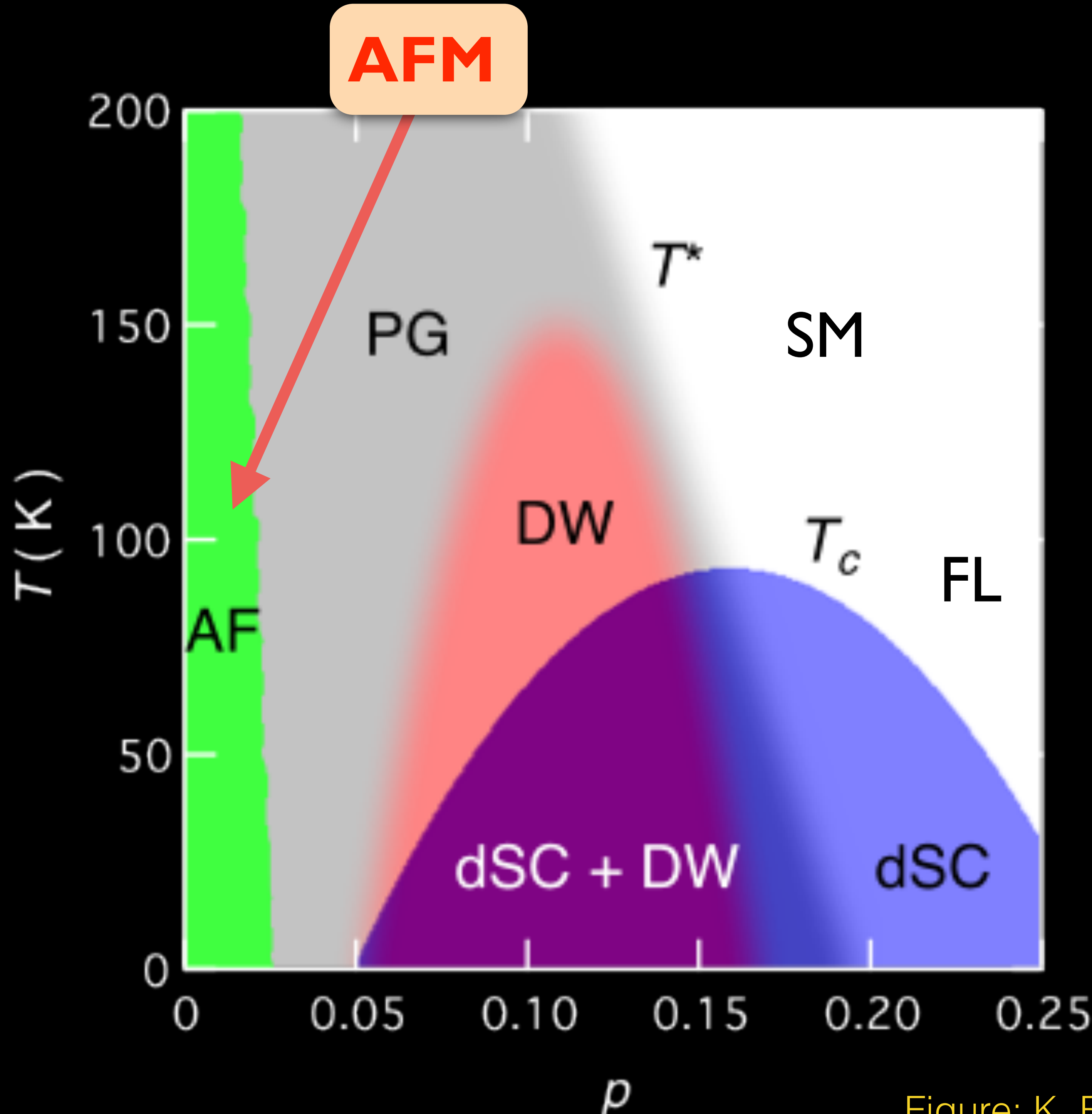
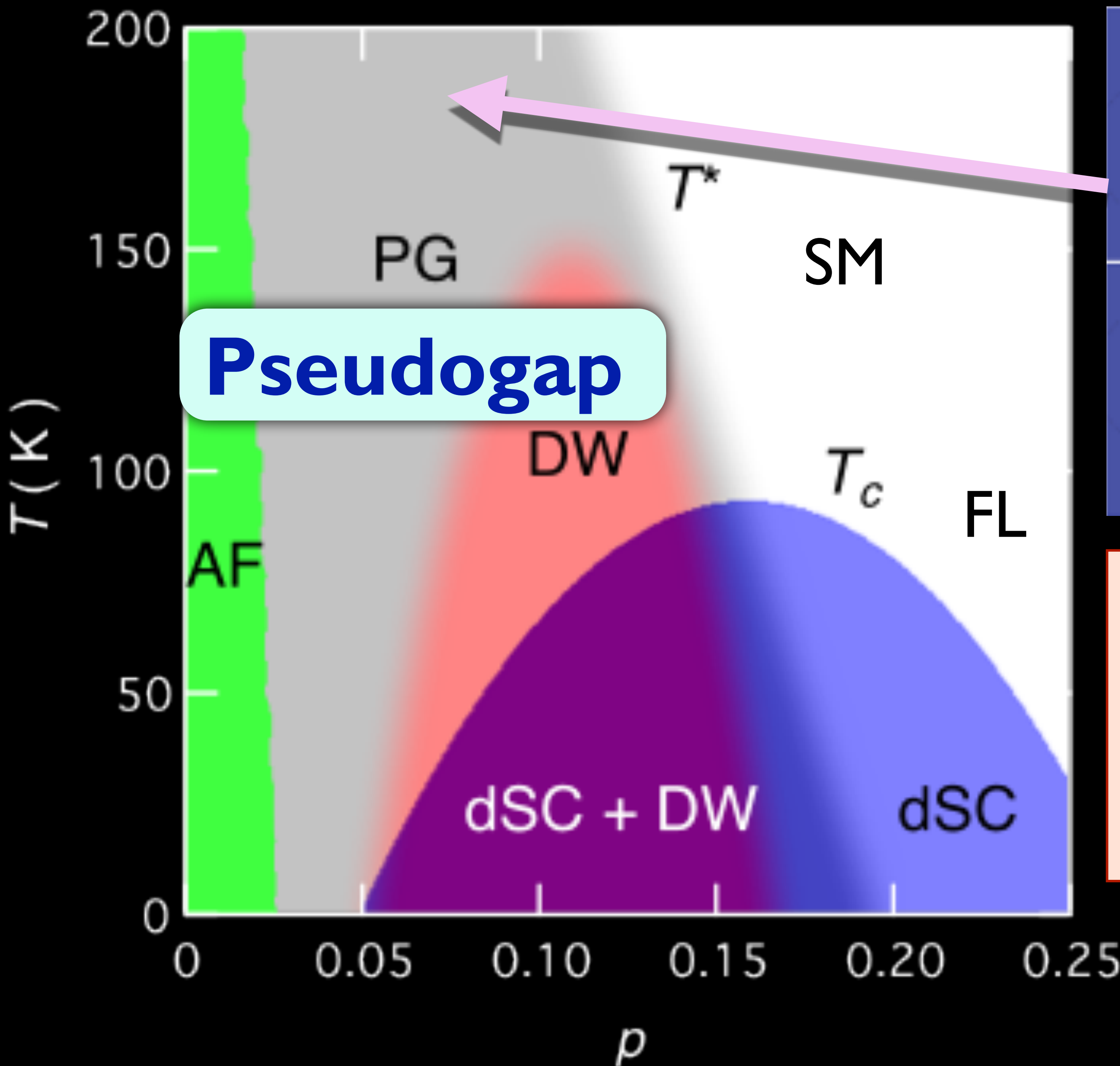
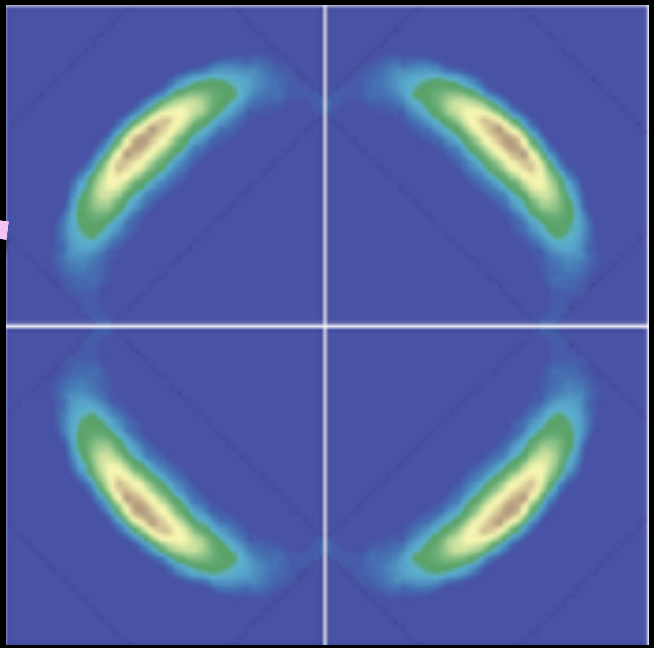


Figure: K. Fujita and J. C. Seamus Davis

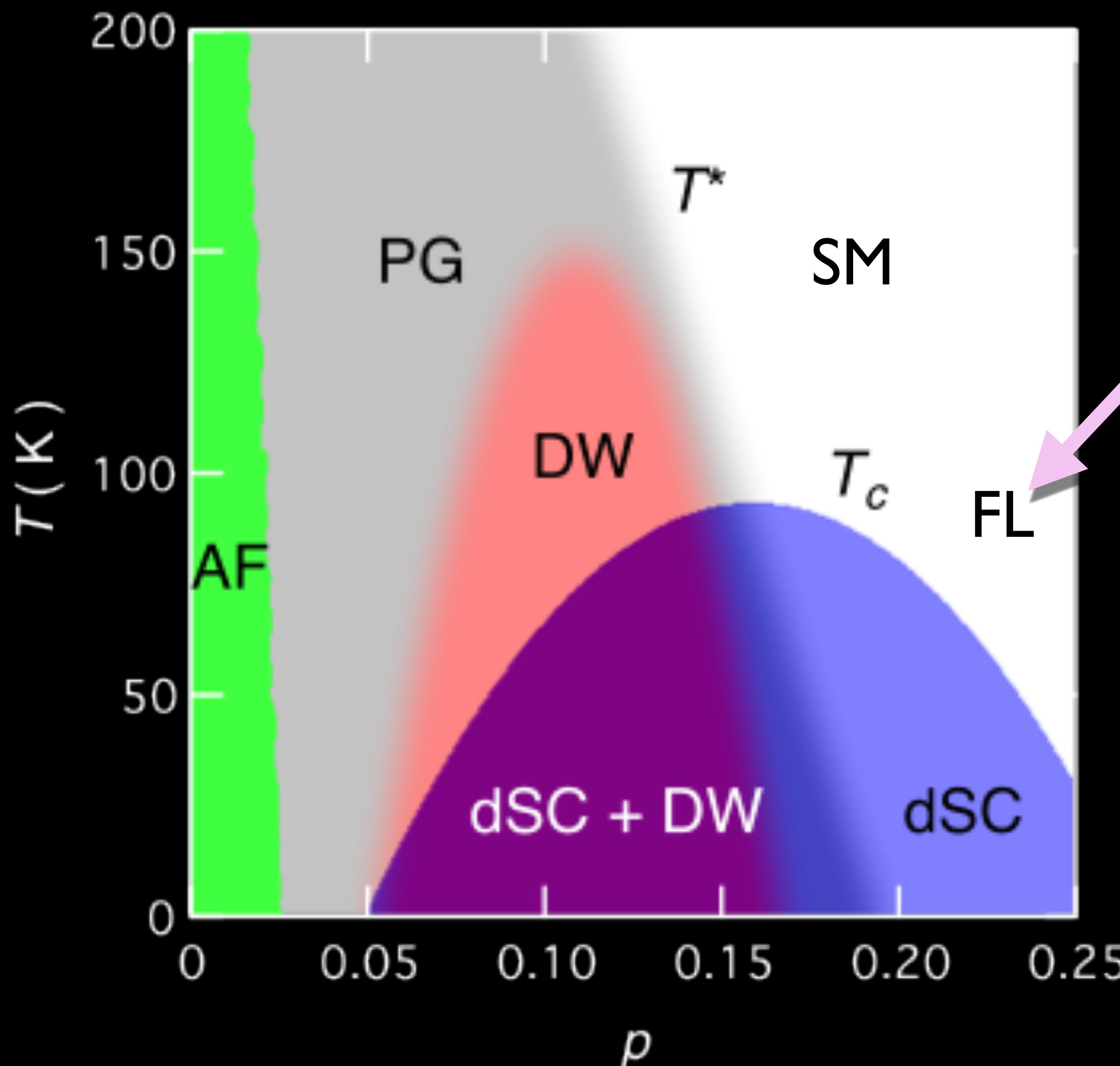
Kyle M. Shen, F. Ronning, D. H. Lu, F. Baumberger, N. J. C. Ingle, W. S. Lee, W. Meevasana, Y. Kohsaka, M. Azuma, M. Takano, H. Takagi, Z.-X. Shen, Science **307**, 901 (2005)

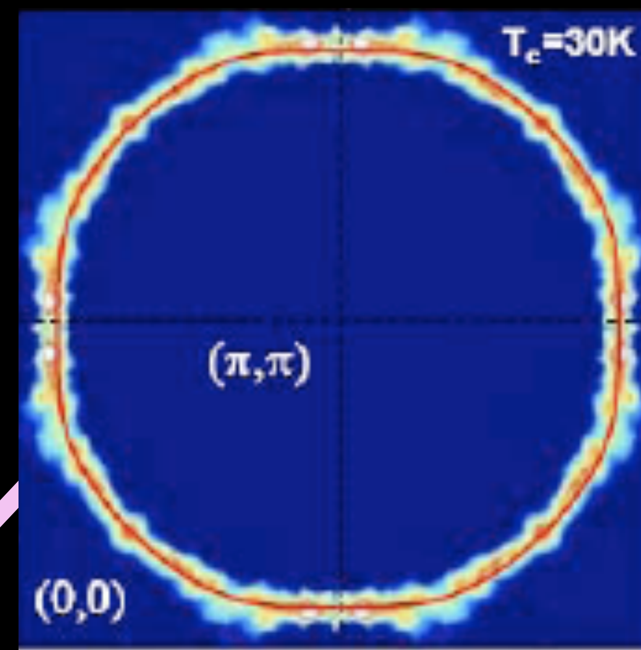




“Fermi arcs”
at
low p

M. Platé, J. D. F. Mottershead, I. S. Elfimov, D. C. Peets, Ruixing Liang, D. A. Bonn, W. N. Hardy, S. Chiuzbaian, M. Falub, M. Shi, L. Patthey, and A. Damascelli, Phys. Rev. Lett. **95**, 077001 (2005)

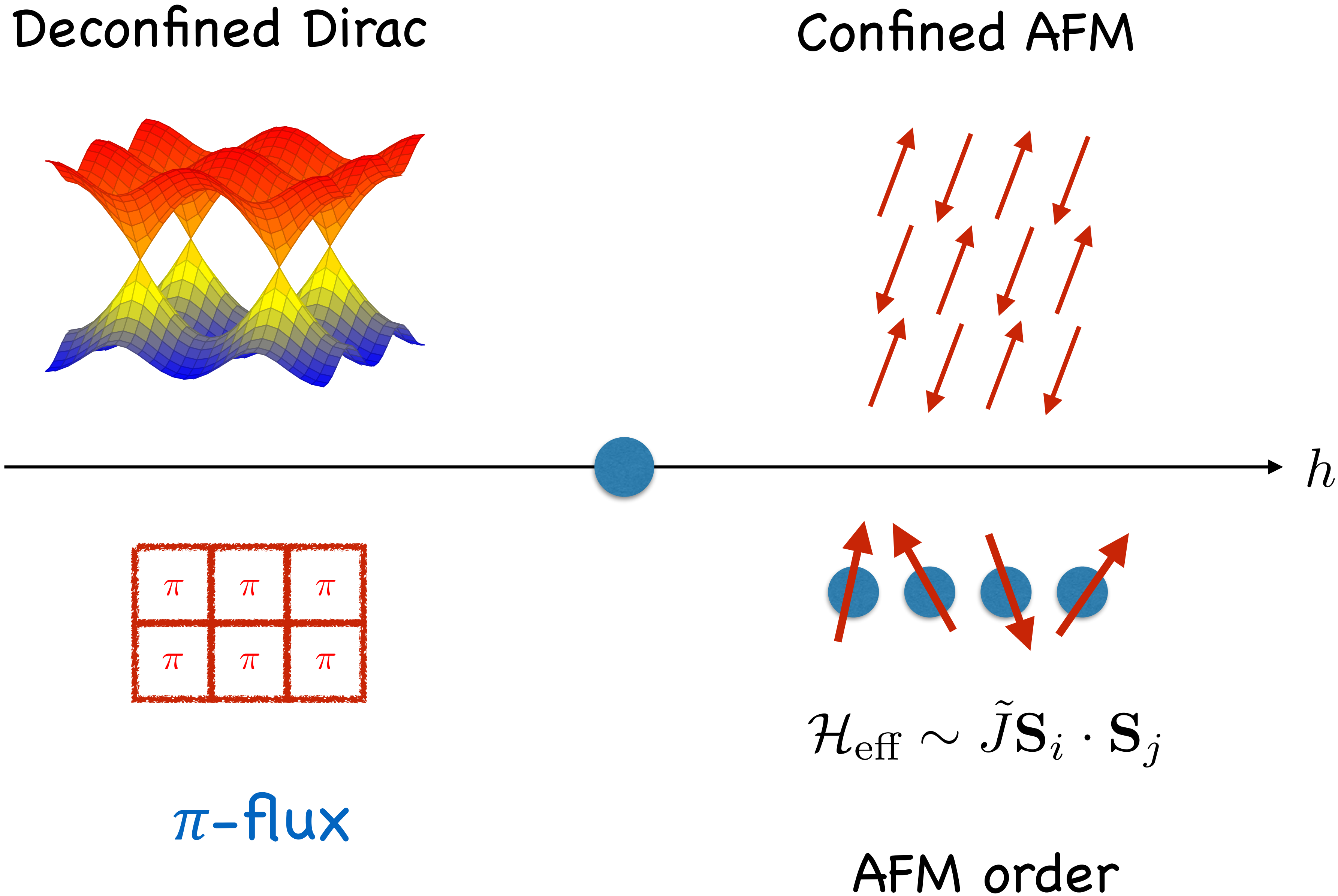




Conventional metal

Area enclosed by Fermi surface $=1+p$

Phase Diagram

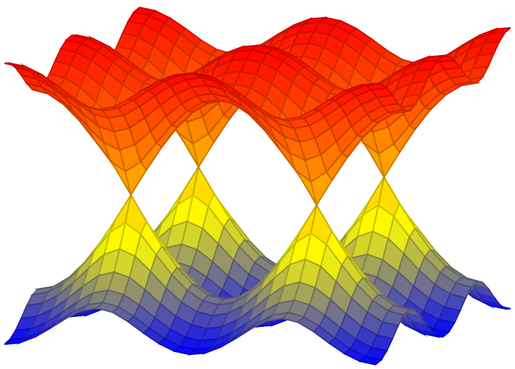


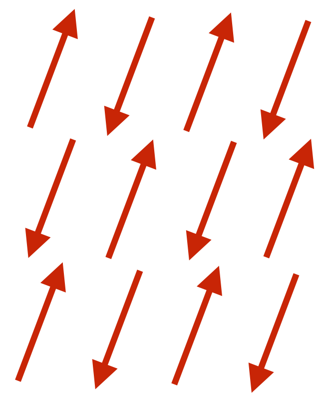
Phase Diagram

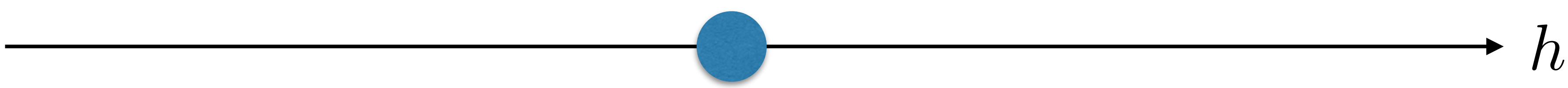
Toy model of pseudogap
(at half filling)

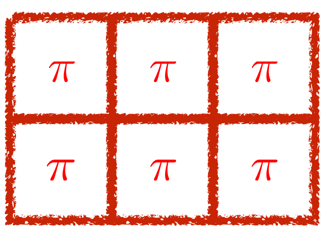
Deconfined Dirac

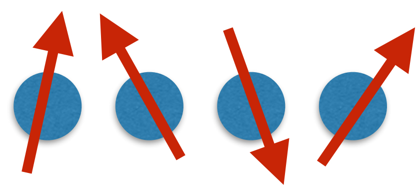
Confined AFM









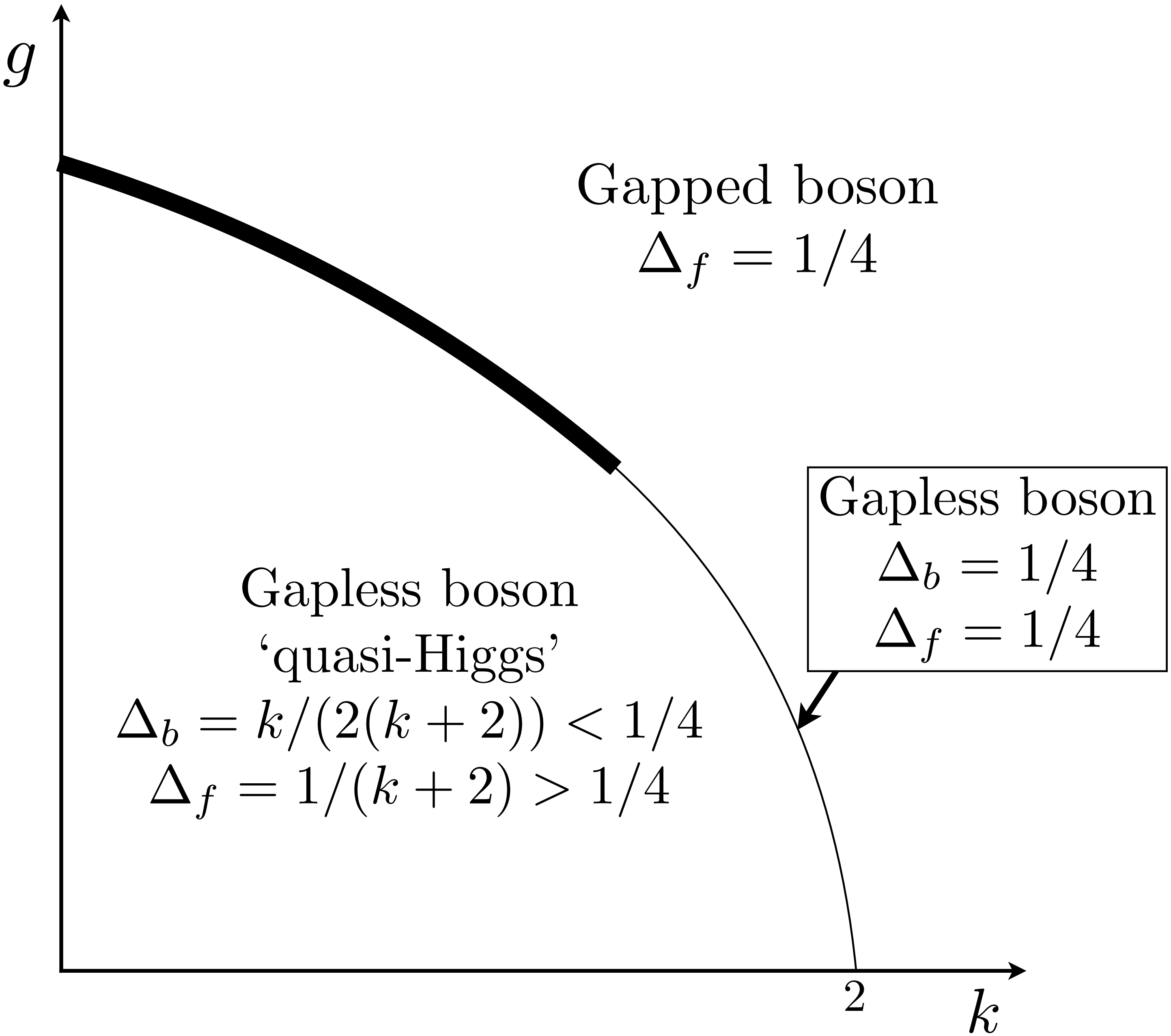


$$\mathcal{H}_{\text{eff}} \sim \tilde{J}\mathbf{S}_i \cdot \mathbf{S}_j$$

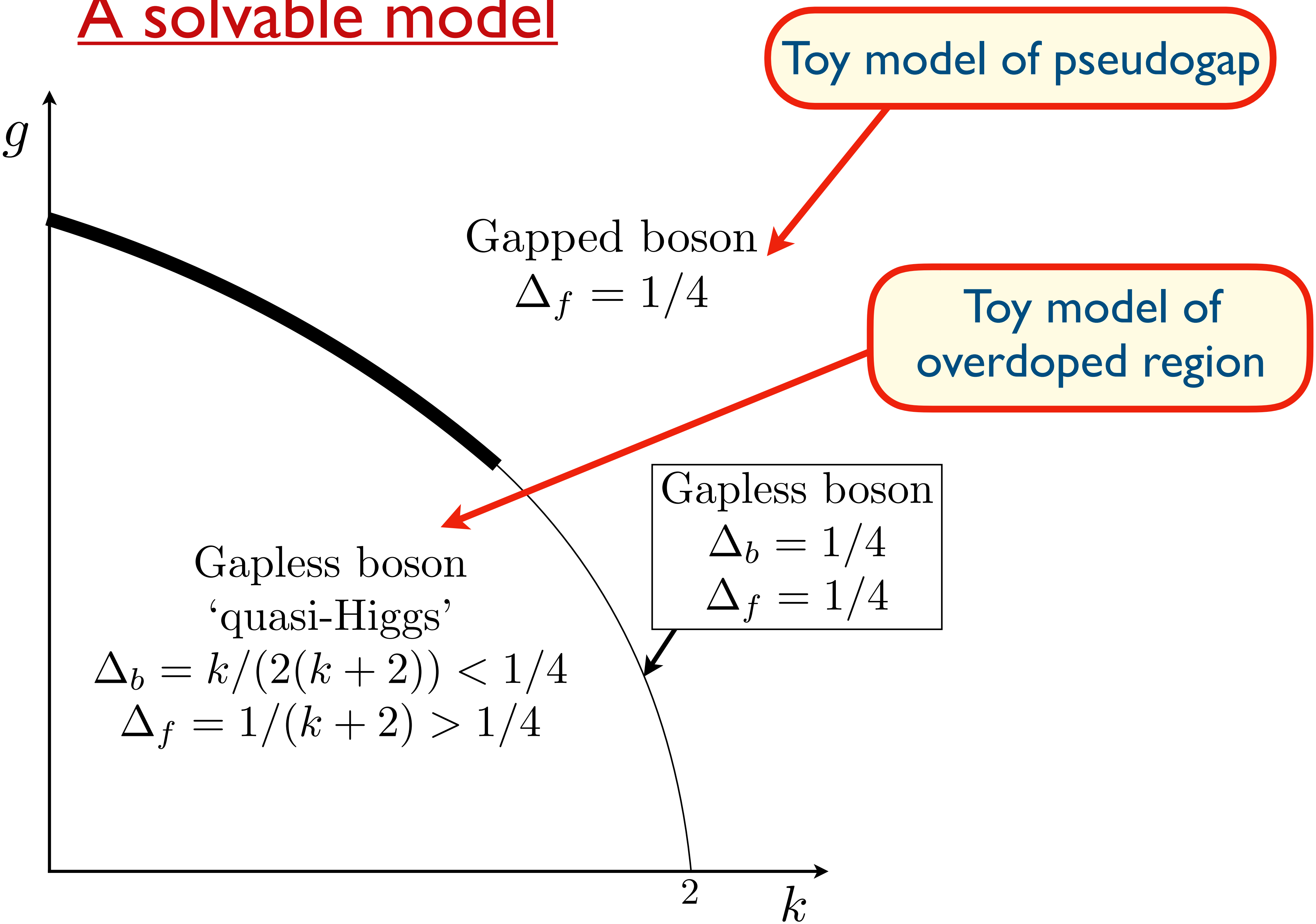
π-flux

AFM order

A solvable model



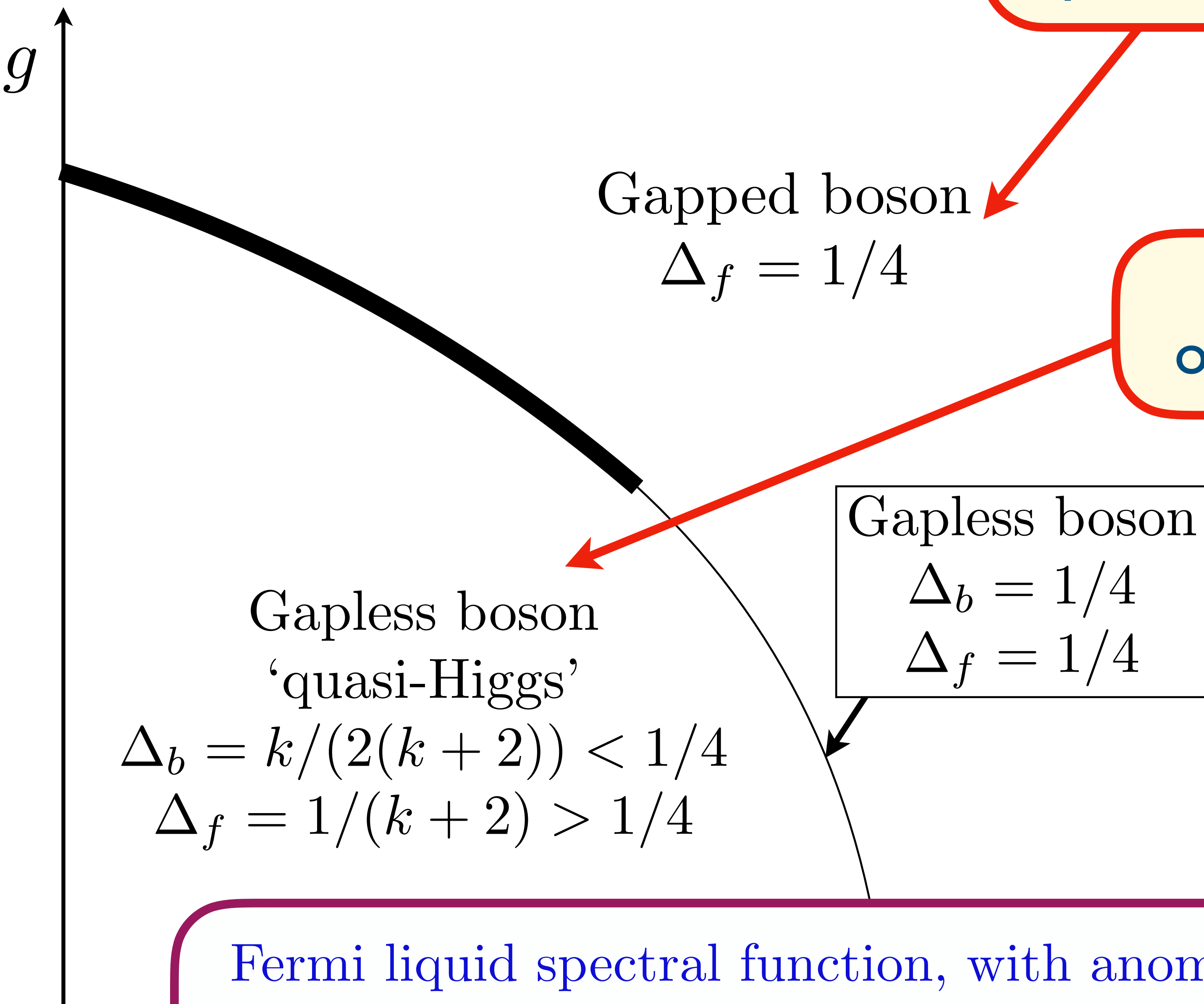
A solvable model



A solvable model

Toy model of pseudogap

Toy model of overdoped region



Fermi liquid spectral function, with anomalies in other properties, match recent observations in cuprates (Hussey, Bozovic, Armitage, Taillefer...)